

Scenarios of cosmic string with variable cosmological constant

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ABSTRACT

Exact solutions of the Einstein field equations with cosmic string and space varying cosmological constant, viz., $\Lambda = \Lambda(r)$, in the energy-momentum tensors are presented. Three cases have been studied: where variable cosmological constant (1) has power law dependence, (2) is proportional to the string fluid density, and (3) is purely a constant. Some cases of interesting physical consequences have been found out such that (i) variable cosmological constant can be represented by a power law of the type $\Lambda = 3r^{-2}$, (ii) variable cosmological constant and cosmic string density are interdependent to each other according to the relation $\Lambda = -8\pi\rho_s$, and (iii) cosmic string density can be scaled by a power law of the type $\rho_s = r^{-2}$. It is also shown that several known solutions can be recovered from the general form of the solutions obtained here.

Key words: relativity – stars : general – stars : interior – cosmology: miscellaneous.

1 INTRODUCTION

The cosmological constant Λ has a fairly long history of acceptance and rejection due to its peculiar properties and behaviour. Historically, it was first proposed by Einstein (1917) as repulsive pressure to achieve a stationary universe for his theory of general relativity. But, later on, theoretical work of Friedmann (1922) without cosmological constant and observational result of Hubble (1929) with galactic law of red-shifts – all did indicate towards an expanding universe paradigm. This situation forced Einstein to abandon the concept of introducing Λ in his gravitational field equations. Thus what was rejected by Einstein the same was embraced by Friedmann and Hubble. This is the first phase for its weird history of ups and downs in the first quarter of nineteenth century.

However, the debate went on with its non-existence and even existence. Actually Zel'Dovich (1968) in his most innovative way revived the issue of cosmological constant Λ by identifying it with the vacuum energy density due to quantum fluctuations. Therefore, people started thinking about Λ once again with a new outlook. If it does exist then what are the features and implications of it on the physical systems are available in the review articles by Weinberg (1989), Carroll, Press and Turner (1992), Sahni and Starobinsky (2000) and Padmanabhan (2003). Following all these what we are really interested to mention here that survival of cosmological constant Λ was occurring slowly and getting gradually a strong theoretical footings. However, the resurrection came in a bold way through the observational evidence of high redshift Type Ia supernovae (Perlmutter 1998; Riess 1998) for a small decreasing value of cosmological constant ($\Lambda_{\text{present}} \leq 10^{-56} \text{ cm}^{-2}$) at the present

epoch. It is now commonly believed by the scientific community that via cosmological constant a kind of repulsive pressure, *dark energy*, is responsible for this present state of accelerating Universe. This is because of the fact that the Λ -CDM model agrees closely with all most all the established cosmological observations, the latest being the 2005 Supernova Legacy Survey (SNLS). According to the first year data set of SNLS *dark energy* behaves like the cosmological constant to a precision of 10 per cent (Astier et al. 2006).

Obviously, this small decreasing value of cosmological constant indicates that instead of a strict constant then Λ could be a function of space and time coordinates or even of both. It is, therefore, argued that the solutions of Einstein's field equations with variable Λ will have a wider range and the roll for scalar Λ in astrophysical problems, specially in relation to the nature of local massive objects like galaxies, will be as much significant as in cosmology (Narlikar et al. 1991; Tiwari, Ray & Bhadra 2000; Ray & Bhadra 2004). In this line of approach, the field equations including Λ have been solved in various cosmological as well as astrophysical situations by several workers (Sakharov 1968; Gunn & Tinsley 1975; Lau 1985; Bertolami 1986a,b; Reuter & Wetterich 1987; Peebles & Ratra 1988; Berman, Som & Gomide 1989; Chen & Wu 1990; Sistero 1991; Kalligas, Wesson & Everitt 1992; Beesham 1993; Tiwari & Ray 1996; Bronnikov, Dobosz & Dymnikova 2003; Wang & Meng 2005; Solà & Štefančič 2006).

Now, string theory being a major tool for understanding nature involving physical systems a string theoretic approach towards astrophysical and cosmological realm thought to be useful from the

very beginning of the advent of the theory. Letelier (1979, 1981) studied the spherical symmetric gravitational field produced by a bunch of strings by replacing the motion of dust clouds and perfect fluids in terms of string clouds and string fluids. Later on Vilenkin (1984) proposed a model of a universe composed of string clouds. Einstein field equations have also been solved by several workers from string theoretic point of view for a single cosmic string (Gott 1985; Hiscock 1985), two moving straight strings (Gott 1991) and N straight strings moving on a circle (Kabat 1992). Dabrowski and Stelmach (1989) considered Friedman universe filled with mutually noninteracting pressureless dust, radiation, cosmological constant and dense system of strings. There are also several other very recent models where the concepts of string have been considered, e.g., rolling tachyon from the string theory (Kremer & Alves 2004), a string-inspired scenario associated with a rolling massive scalar field on D-branes (Garousi, Sami & Tsujikawa 2004), exotic matter, either in the form of cosmic strings or struts with negative energy density also has been considered in connection to the matter content of an additional thin rotating galactic disk (Vogt & Letelier 2005) and a fluid of strings along with the cosmological constant, as a candidate of dark energy, has considered by Capozziello et al. (2006) to get a viable scenario of the present status of the Universe.

So, it is not unnatural to investigate the effect of Λ on an anisotropic static spherically symmetric source composed of string fluid. Therefore, the present investigation is the generalization of the work of Tiwari and Ray (1996) with string fluid replacing perfect fluid. The main purpose of the investigations are: firstly, to search for some kind of link between the approaches with matter and string theoretic one; secondly, interrelationship, if exist, between the string and cosmological constant; and thirdly, to find out the extra features which appear due to the inclusion of varying cosmological constant in string fluid.

The paper is organized as follows: in Section 2 the general relativistic Einstein field equations are presented where the cosmological constant and string fluid are included in the energy-momentum tensors. Section 3 deals with their solutions in various cases and subcases under this modified theory of general relativity. Some salient features of the present model are discussed in the concluding Section 4.

2 EINSTEIN FIELD EQUATIONS

The Einstein field equations are given by

$$G^i_j = R^i_j - \frac{1}{2}g^i_j R = -8\pi T^i_j - g^i_j \Lambda, \quad (i, j = 0, 1, 2, 3) \quad (1)$$

where in relativistic units both G and c of $\kappa (= 8\pi G/c^4)$ are equal to 1. Here, we shall assume the cosmological constant Λ to be space-varying, i.e., $\Lambda = \Lambda(r)$. It is to be noted here that the left hand side of the Einstein field equations express the geometrical structure of the space-time whereas the right hand side is the representative of the matter content. We have, therefore, purposely put the non-zero Λ term in the right to make its status as physical one such that the conservation law in the present case takes the form as follows (Peebles & Ratra, 1988)

$$8\pi T^i_j{}_{;i} = -\Lambda_{;j}. \quad (2)$$

Let us consider a spherically symmetric line element

$$ds^2 = g_{ij}dx^i dx^j = e^{\nu(r)} dt^2 - e^{\lambda(r)} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (3)$$

where ν and λ are the metric potentials and are function of the space coordinate r only, such that $\nu = \nu(r)$ and $\lambda = \lambda(r)$.

Now, if we assume that (1) the fluid source is a perfect fluid of finite-length straight strings and (2) the individual strings do orient themselves in a perfectly radial direction, then the energy-momentum tensors reduce to

$$T^t_t = T^r_r \quad \text{and} \quad T^\omega_\omega = q \quad (4)$$

where ω stands for both the angular coordinates θ and ϕ related to the metric (3). In this context it is to be noted here that the above assumptions are quite relevant to the results of Gott (1985) and Hiscock (1985) that a straight ideal string has vanishing gravitational mass due to the gravitational effect of tension which exactly cancels the effect of mass. Therefore, strings do not produce any gravitational force and can be thought of exactly straight and radially oriented (Letelier 1979; Soleng 1995).

Now, to solve the Einstein's gravitational field equations uniquely one has to specify some relationship between the energy-momentum tensors. Let us, therefore, assume that the transverse pressure part of the string energy-momentum tensor T^t_t is proportional to the angular part of the energy-momentum tensors T^ω_ω . Then, from the above equation (4), we get

$$T^t_t = T^r_r = -\alpha T^\omega_\omega \quad (5)$$

where α is a dimensionless constant of proportionality. This assumption (5), as mentioned in the previous discussion, indicates the perfect radial orientation of the strings. Here, however, we would like to mention that this algebraic form of energy-momentum tensors was earlier considered by Gliner (1966) and Petrov (1969) in mathematical context and later on by Dymnikova (1992) and Soleng (1994) associated with anisotropic vacuum polarization in a spherically symmetric space-time.

By virtue of the equations (3) and (5), the field equations (1) can explicitly be expressed as

$$e^{-\lambda} \left(\frac{\lambda'}{r} - \frac{1}{r^2} \right) + \frac{1}{r^2} = 8\pi\rho_s + \Lambda, \quad (6)$$

$$e^{-\lambda} \left(\frac{\nu'}{r} + \frac{1}{r^2} \right) - \frac{1}{r^2} = -8\pi\rho_s - \Lambda, \quad (7)$$

$$e^{-\lambda} \left[\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\nu'\lambda'}{4} + \frac{(\nu' - \lambda')}{2r} \right] = \frac{8\pi}{\alpha}\rho_s - \Lambda. \quad (8)$$

The above field equations, at a glance, give some clue in between the metric potentials. Therefore, addition of the equations (6) and (7) immediately provides us the following relation

$$\nu = -\lambda \quad (9)$$

which is equivalent to $g_{00}g_{11} = -1$, between the metric potentials of the metric equation (3). A coordinate-independent statement of this relation have been obtained by Tiwari, Rao and Kanakamedala (1984) by using the eigen values of the Einstein tensor G^i_j expressed in equation (1). It is also interesting to note that $g_{00}g_{11} = -1$ can be expressed in terms of the energy-momentum tensors $T^1_1 = T^0_0$.

The energy conservation law, in the present situation, is given by

$$\frac{d}{dr} \left[-\rho_s - \frac{\Lambda}{8\pi} \right] = \frac{2}{r} \left[\rho_s + \frac{\rho_s}{\alpha} \right] = \frac{\beta}{r} \rho_s \quad (10)$$

where

$$\beta = 2 \left(1 + \frac{1}{\alpha} \right). \quad (11)$$

Then, from the equation (10), we get

$$\frac{d\rho_s}{dr} + \frac{\beta}{r}\rho_s = -\frac{1}{8\pi} \frac{d\Lambda}{dr} \quad (12)$$

which, on multiplication by r^β and then integrating it, reduces to

$$\rho_s r^\beta = \frac{A}{8\pi} - \frac{1}{8\pi} \int \Lambda' r^\beta dr \quad (13)$$

where $A/8\pi$ is an integration constant.

3 THE SOLUTIONS

To obtain some explicit simplified results, let us investigate the field equations under the following special assumptions.

3.1 The case for $\Lambda' \propto r^{-\beta}$

Let us assume that

$$\Lambda' = Br^{-\beta} \quad (14)$$

where B is a proportional constant.

By the use of this assumption (14), we get from the equation (13)

$$\rho_s r^\beta = \frac{A}{8\pi} - \frac{1}{8\pi} \int Bdr, \quad (15)$$

so that the energy density can easily be obtained as

$$\rho_s = \frac{A}{8\pi} r^{-\beta} - \frac{B}{8\pi} r^{1-\beta}. \quad (16)$$

Again, on integration, equation (14) yields

$$\Lambda = \frac{B}{1-\beta} r^{1-\beta}. \quad (17)$$

Using the equations (16) and (17) we get from the equation (6)

$$e^{-\lambda} \left(\frac{\lambda'}{r} - \frac{1}{r^2} \right) + \frac{1}{r^2} = Ar^{-\beta} + \frac{B}{1-\beta} r^{1-\beta} - Br^{1-\beta}, \quad (18)$$

so that after simplification it takes the following differential form

$$d(re^{-\lambda}) = dr - Ar^{2-\beta} dr + Br^{3-\beta} \left[1 - \frac{1}{1-\beta} \right] dr. \quad (19)$$

The above equation (19) on integration provides the form for the metric potentials as

$$e^{-\lambda} = 1 - \frac{2M}{r} - \frac{A}{3-\beta} r^{2-\beta} - \frac{B\beta}{(1-\beta)(4-\beta)} r^{3-\beta} = e^\nu \quad (20)$$

where $2M$ is an integration constant.

Thus the general form of the space-time can be given by

$$ds^2 = \left[1 - \frac{2M}{r} - \frac{A}{3-\beta} r^{2-\beta} - \frac{B\beta}{(1-\beta)(4-\beta)} r^{3-\beta} \right] dt^2 - \left[1 - \frac{2M}{r} - \frac{A}{3-\beta} r^{2-\beta} - \frac{B\beta}{(1-\beta)(4-\beta)} r^{3-\beta} \right]^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (21)$$

3.1.1 $\alpha = -2$

For this subcase when $\beta = 1$ readily makes Λ , via equation (17), to be infinite and hence is not a well-defined quantity. However, for $\beta = -1$ one can get

$$\Lambda = \frac{B}{2} r^2. \quad (22)$$

For a particular choice of the values for $\beta = 3$ and $B = 1$, therefore, equation (17) gives

$$\Lambda = \frac{3}{r^2}. \quad (23)$$

This result is similar to the one as obtained by Krisch and Glass (2003) for $B = -6$ when r is recognized as the cosmic scale factor in connection to a toroidal fluid solution embedded in a locally anti-de Sitter exterior.

3.1.2 $\alpha = -1$

For this, we get from the equation (11) the value for β as zero, which reduces the equation (10) in the form

$$\frac{d}{dr} \left[\rho_s + \frac{\Lambda}{8\pi} \right] = 0. \quad (24)$$

This immediately yields

$$\rho_s + \frac{\Lambda}{8\pi} = C \quad (25)$$

where C is an integration constant. Hence, from the equation (6), after integrating it and using results of (9) and (25), we get

$$e^{-\lambda} = 1 - \frac{2M}{r} - \frac{8\pi C}{3} r^2 = e^\nu. \quad (26)$$

From the above equation (26) we find that, for the appropriate choice of value of the constant as $C = 0$, it turns out to be Schwarzschild solution. A comparison of the equations (20) and (26), via equation (25), immediately reveals that $A = 8\pi C = \Lambda + 8\pi\rho_s$ for $\beta = 0$. Therefore, choosing suitably the values of $A = C = 0$ one can obtain

$$\Lambda = -8\pi\rho_s. \quad (27)$$

This means that, even in a restricted case, vacuum energy density is dependent on cosmic string density and vice versa. It is worthwhile to note that, in the context of the inflationary cosmology (Guth 1981; Sato 1981; Linde 1982; Albrecht & Steinhardt 1982; Linde 1983), the above relation (27) can be expressed as $p_s = -\rho_s$, where p_s , now equivalent to $\Lambda/8\pi$, is the fluid pressure of string. The equation of state of this type $p_s = \gamma\rho_s$ with $\gamma = -1$ implies that the matter distribution is in tension and hence the matter is known, in the literature, as a 'false vacuum' or 'degenerate vacuum' or ' ρ -vacuum' (Davies, 1984; Blome & Priester, 1984; Hogan, 1984; Kaiser & Stebbins, 1984).

Now two cases may be considered here –

Case I: When the condition is $\rho_s > 0$ and $\Lambda < 0$, we get the positive energy density of string with negative vacuum energy density. We emphasize here that in the cosmological context Λ positive is related to the repulsive pressure. Λ being negative, as evident from the equation (27), this case does not correspond to the present state of the accelerating Universe rather it is related to a collapsing situation (Cardenas et al. 2002).

Case II: When the condition is $\rho_s < 0$ and $\Lambda > 0$, we get the negative energy density of string with positive vacuum energy density. This case of positive Λ and hence negative pressure, therefore,

indicates towards a state of acceleration. This positive Λ as appears in the form of the negative pressure try to expand the space-time curvature in the outward direction and thus does play the role for *dark energy* which is responsible for the present status of the cosmic acceleration (Perlmutter, 1998; Riess, 1998). It is to be noted here that the concept of negative energy density of string is not unrealistic in the realm of string theory and have been extensively studied by several workers (Sen & Zwiebach 2000; Vollick 2002; Horowitz 2003; Hertog, Horowitz & Maeda 2003, 2004).

3.2 The case for $\Lambda \propto \rho_s$

Let us assume here that

$$\Lambda = 8\pi D \rho_s \quad (28)$$

where D is a constant of proportionality.

From the above assumption (28), after differentiating it, we get

$$\frac{d\Lambda}{dr} = 8\pi D \frac{d\rho_s}{dr} \quad (29)$$

for which the equation (12) reduces to

$$\frac{d\rho_s}{dr} + \frac{\beta}{r} \rho_s = -D \frac{d\rho_s}{dr}. \quad (30)$$

On integration this yields

$$\rho_s = D_0 r^{-\beta/(1+D)}. \quad (31)$$

Therefore, by the use of the equations (28) and (31), we get from the equation (6)

$$e^{-\lambda} = 1 - \frac{2M}{r} - \frac{8\pi D_0(1+D)^2}{3+3D-\beta} r^{(2+2D-\beta)/(1+D)} = e^\nu. \quad (32)$$

Thus the general form of the space-time can be written as

$$ds^2 = \left[1 - \frac{2M}{r} - \frac{8\pi D_0(1+D)^2}{3+3D-\beta} r^{(2+2D-\beta)/(1+D)} \right] dt^2 - \left[1 - \frac{2M}{r} - \frac{8\pi D_0(1+D)^2}{3+3D-\beta} r^{(2+2D-\beta)/(1+D)} \right]^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (33)$$

3.2.1 $D = -1$

This subcase, obviously, goes back to the subcase 3.1.1 for $A = C = 0$ so that the explanation in connection to the negative string density and cosmic acceleration is also valid here. The Schwarzschild vacuum solution also can be recovered here as a special case in the form given by

$$ds^2 = \left[1 - \frac{2M}{r} \right] dt^2 - \left[1 - \frac{2M}{r} \right]^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (34)$$

This space-time is associated with a particle of mass M centered at the origin of the spherical system surrounded by a cloud of strings of density $\rho_s = D_0 r^{-\infty}$, for $D_0 \neq 0$. Obviously, there does not exist any string around the spherical configuration except at the points where $r = \pm 1$. At $r = 0$ the string density and space-time both blow up and thus *singularity* arises in the solution. According to Soleng (1995), however, this singularity at the origin is not a unique feature for string fluid model the mass being surrounded by a string fluid.

3.2.2 $D = 0$

For this subcase the cosmological parameter Λ vanishes by virtue of assumption (28) and hence the equation (31) reduces to

$$\rho_s = D_0 r^{-\beta}. \quad (35)$$

This result can easily be recognized as the one obtained by Turok and Bhattacharjee (1984) and also others (Kibble 1986; Vachaspati & Vilenkin 1987) where $2 \leq \beta \leq 3$ in their case. If we now choose $\alpha \gg 1$ then for this very large value of α , when $\beta = 2$, the equation (31) becomes

$$\rho_s \propto r^{-2}. \quad (36)$$

This suggests that string-dominated universe expands more rapidly than matter- or radiation-dominated one where energy density varies, respectively, as $\rho_m \propto r^{-3}$ and $\rho_r \propto r^{-4}$. This case $\beta = 2$ and the astronomical constraints on string-dominated universe have been investigated by Gott and Rees (1987). However, Dabrowski and Stelmach (1989) prefer this case $\beta = 2$ as it corresponds to the set of randomly oriented straight strings or to the tangled network of strings which conformally stretches by expansion. According to them unlike $2 \leq \beta \leq 3$ this case is particularly interesting because it allows treatment of all type of components of the universe simultaneously in an analytic way and also gives some aspects of observational problems in the universe with strings. Vilenkin (1985) and Soleng (1995) also exploit this type of string cloud with energy density $\rho_s \propto r^{-2}$.

In this connection we are interested to point out that the present model with $\rho_s \propto r^{-\beta}$ is more general than others as mentioned above and can be applicable, at least theoretically, with out any constrains like $2 \leq \beta \leq 3$. Another point to note here is that this result appears in the form of a solution of the Einstein field equations not as an *ad hoc* assumption.

3.3 The case for $\Lambda = \text{constant}$

For this trivial case of erstwhile cosmological constant, the equation (13) becomes

$$\rho_s = \frac{A}{8\pi} r^{-\beta}. \quad (37)$$

This is again the same relation (35) for $A = 8\pi D_0$. Therefore, we can recover the relation $\rho_s \propto r^{-2}$ from here also for $\alpha = \infty$, $\beta = 2$ and hence the explanation of this can be made in a similar way as before.

By the use of above expression in the equation (6) the metric potential can now be given by

$$e^{-\lambda} = 1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2 - \frac{A}{3-\beta} r^{2-\beta} = e^\nu. \quad (38)$$

Thus the general form of the space-time can be written as

$$ds^2 = \left[1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2 - \frac{A}{3-\beta} r^{2-\beta} \right] dt^2 - \left[1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2 - \frac{A}{3-\beta} r^{2-\beta} \right]^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (39)$$

3.3.1 $\alpha = -1$

For this subcase we get $\beta = 0$ and hence the equation (38) reduces to

$$e^{-\lambda} = 1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2 - \frac{A}{3}r^2 = e^{\nu}. \quad (40)$$

One can observe that for $A = 0$ the above space-time becomes the Schwarzschild-de Sitter vacuum solution which again, in the absence of Λ reduces to the Schwarzschild vacuum solution as in the case of Soleng (1995) for $\alpha = -1$ and 0 respectively. Moreover, in this case the string density becomes a constant quantity as given by

$$\rho_s = \frac{A}{8\pi} \quad (41)$$

which means that the space-time associated with the particle of mass M is surrounded by a spherical cloud of constant string density here.

3.3.2 $\alpha = 1$

In this subcase of $\beta = 4$, we get the metric as

$$ds^2 = \left[1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2 + \frac{A}{r^2}\right] dt^2 - \left[1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2 + \frac{A}{r^2}\right]^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (42)$$

If we now smoothly match this space-time with that of the Reissner-Nordström on the boundary of the spherical object of radius R such that $R = r$, then we get

$$A = Q^2 \quad (43)$$

in the absence of Λ . Therefore, the constant A can easily be identified with the square of the charge contained in the sphere and hence the above solution provides a Reissner-Nordström black hole surrounded by strings of fluid density $\rho_s = (A/8\pi)r^{-4}$. However, in the case of $\Lambda \neq 0$ this turns into a Reissner-Nordström-de Sitter universe (Soleng 1995).

4 CONCLUSIONS

Our three major results in the present investigations are as follows:

Firstly, it is observed that variable cosmological constant has power law dependence in the form $\Lambda = 3r^{-2}$ for a particular value for $\beta = 3$. This gives the toroidal fluid solution embedded in a locally anti-de Sitter exterior (Krisch & Glass 2003).

Secondly, it is possible to show that by the suitable choice of the values of the constants one can easily obtain the relation $\Lambda = -8\pi\rho_s$. This means that locally vacuum energy density and cosmic string density are interdependent to each other. It is already known that cosmological constant is responsible for providing repulsive pressure to the present state of accelerating Universe. Therefore, one can explore the possibilities of whether there is any relation between the cosmic acceleration and cosmic string.

Thirdly, it is shown that cosmic string density can be scaled by a power law of the type $\rho_s \propto r^{-2}$ (Turok & Bhattacharjee 1984; Kibble 1986; Vachaspati & Vilenkin 1987; Gott & Rees 1987; Dabrowski & Stelmach 1989). This result indicates that string-dominated universe expands more rapidly than matter-dominated universe ($\rho_m \propto r^{-3}$) or radiation-dominated universe ($\rho_r \propto r^{-4}$).

It is also observed that from the general solutions several known solutions such as the Schwarzschild vacuum solution, the Schwarzschild-de Sitter vacuum solution, the Reissner-Nordström space-time and the Reissner-Nordström-de Sitter universe can be recovered here (Soleng 1995).

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